# The Acceleration of the Solar System Barycenter Determined by Very Long Baseline Interferometry

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#### Abstract

The secular aberration drift observed as the apparent proper motion of extragalactic radio sources is caused by the acceleration of the coordinate origin, the solar system barycenter. Therefore, the three-dimensional vector of the acceleration of the solar system barycenter was estimated as a global parameter by using the 30 years of global geodetic and astrometric VLBI data. This estimation is independent of any kinematic or dynamic model of the Milky Way or statistics hypothesis. The estimated acceleration in the direction of the Galactic center is  $7.47 \pm 0.46 \text{ mm} \cdot s^{-1} \cdot yr^{-1}$ , while the estimation of the other two components are  $0.17 \pm 0.57$  and  $3.95 \pm 0.47 \text{ mm} \cdot s^{-1} \cdot yr^{-1}$  in the direction along the Solar motion in the Galactic plane and in the direction normal to the Galactic plane, respectively. The estimate of the acceleration independently provides another kinematics parameter without referring to any objects within the Galaxy.

## 1. Introduction

The definition, realization, and maintenance of a celestial reference system is one of the oldest and most fundamental tasks in astronomy. The previous celestial reference frame was constituted in the optical regime by the Fifth Fundamental Catalogue FK5 [1], which was a practical realization by mean positions and proper motions of a group of fundamental stars. Owing to the fact that the FK5 frame is based on the theory of the motions of bodies in the solar system and the assumptions of the proper motions of stars close to the Sun, it has a hybrid definition, dynamic and kinematic. With the advent of the most precise astrometry conducted by the technique of Very Long Baseline Interferometry (VLBI), uncertainties of radio source positions achieved the level of less than one milliarcsecond over most of the sky by the mid 1990s. At the beginning of 1988, a number of International Astronomy Union (IAU) working groups thus initiated consideration of a new concept of the fundamental celestial reference frame [2, 3], and in 1991 IAU recommended that its celestial reference frame should show no global rotation with respect to a set of distant extragalactic objects. However, because it is based on the kinematics of the extragalactic objects with negligible proper motions, it becomes a purely kinematic definition, sometimes also called static [4]. There are two main advantages of this new reference frame: (1) extragalactic radio sources with undetectable proper motions are at rest with respect to the Universe, and (2) the definition of the ICRF is not related to the dynamic equinox.

Nevertheless, while the solar system moves around the Galactic center, its velocity vector is still changing by a tiny amount, which causes the secular aberration drift in radio source directions. For 30 years, this effect will accumulate up to about 200 microarcseconds ( $\mu$ as), which is far beyond the declared several tens of  $\mu$ as level of the ICRF2 accuracy [5] and must be considered. In this paper, we present the result of the solar acceleration obtained by VLBI data analysis.

## 2. Model

The direction of the observed radio source  $\vec{K}$  was always treated as time-independent in the Barycentric Celestial Reference System (BCRS), which is a basic assumption of realizing the celestial reference system by radio source [6, 7]. When considering the effect of the secular aberration drift, the positions of radio sources should present variations in time with a dipolar structure. Then at the arbitrary observing epoch t, the direction of the radio source should be a function of the solar system barycenter's acceleration  $\vec{a}$ , given by

$$\vec{K_t} = \vec{K_0} + \frac{(\vec{K_0} \times \vec{a}) \times \vec{K_0}}{c} (t - t_0),\tag{1}$$

where  $\vec{K_0}$  is the direction of the radio source in the BCRS at the reference epoch  $t_0$  (e.g., J2000.0), and c is the speed of light in a vacuum. The acceleration vector  $\vec{a}$  could be treated as a constant during the approximately 30 year history of VLBI observations.

Based on the formula of the geometric delay given by the IERS 2010 conventions [8], the partial derivative of the VLBI geometric delay  $\tau$  with respect to the instant direction  $\vec{K}_t$  is accurate up to terms of order  $O((\vec{V_{\oplus}} + \vec{\omega_2})^2/c^2)$ ,

$$\frac{\partial \tau}{\partial \vec{K_t}} = -\frac{\vec{b}}{c + \vec{K_t}(V_{\oplus} + \vec{\omega_2})} + \frac{\vec{K_t} \cdot \vec{b}}{c^2} (\vec{V_{\oplus}} + \vec{\omega_2}), \tag{2}$$

where  $\vec{b}$  is the geocentric baseline vector, and  $\vec{V_{\oplus}}$  and  $\vec{\omega_2}$  are the barycentric velocity of the geocenter and the geocentric velocity of the second receiver, respectively. In fact, it is accurate enough to apply  $\vec{K_0}$  instead of the  $\vec{K_t}$  to the right of Equation 2 when calculating the value of this partial derivative.

Given these equations, the partial derivatives with respect to the acceleration are found to be,

$$\frac{\partial \tau}{\partial \vec{a}} = \frac{\partial \tau}{\partial \vec{K}_t} \cdot \frac{\partial \vec{K}_t}{\partial \vec{a}},\tag{3}$$

there is an approach to estimate the solar acceleration vector as a global parameter in the data analysis based on Equation 3, which we call the global solution [9].

## 3. Data and Results

The data we used in our solutions were taken from an International VLBI Service for Geodesy and Astrometry [10] data center. There were 4632 sessions from April 1980 to October 2011. The sessions with durations less than 18 hours were excluded, and the sessions in which the longest baseline length was shorter than 3000 km (small networks) were also excluded. The radio sources with fewer than four good observations in the whole data set were not used, and the data which the station TSUKUB32 observed after the big earthquake in March 2011 were excluded as well. The elevation cut-off angle was set to 7°.

The CALC/SOLVE software developed and maintained by NASA Goddard Space Flight Center was used for the VLBI data analysis. The USER\_PARTIAL function, an external interface provided by this software, was utilized for the integration of the estimation models of the acceleration. Parameters were divided into three groups including:

- 1. Global (over the entire data set): positions and velocities of the stations at the reference epoch; the axis offsets of the antennas; the coefficients of the diurnal, semi-diurnal, annual, and semi-annual tidal model (amplitude and phase) of 41 stations [11], and coefficients for the expansion into B-spline basis of positions of three stations, HRAS\_085, PIETOWN, and GILCREEK;
- 2. Local (over each session): Earth orientation parameters, including polar motion, UT1, and their rates along with the adjustments of two nutation parameters;
- 3. Segmented (over intervals ranging from 20 minutes to 6 hours): the clock offsets and rates (1-hour segments); linear spline coefficients modeling zenith wet delay (20-minute segments); piecewise coefficients of the north and east troposphere gradients (6-hour segments).

In the global solution, the acceleration vector was estimated as a global parameter. The 39 special handling sources were estimated as local parameters, while the positions of the other radio sources were estimated as global parameters. No-net rotation constraints were imposed on the 295 ICRF2 defining sources with respect to their ICRF2 positions. No-net-rotation and no-net-translation constraints on the adjustments of the station positions and velocities of 35 sites with respect to VTRF2008 [12] were applied to realize the earth reference frame. The a priori models for geophysical effects and precession/nutation were based on the IERS2003 conventions. NMF [13] was chosen as the atmosphere mapping function, and the atmospheric pressure loading model [14] and the thermal expansion model of the radio telescopes [15] were also applied. Parameterization is discussed in Xu et al. [16] in more detail.

For all 4632 sessions, 3492 radio sources and approximately 7,100,000 group delay observables were analyzed. In the global solution, the overall post-fit weighted root mean square delay residual has 21.3 ps, and the chi-square has 0.98. In the Galactic Cartesian coordinate system [17, 18], the acceleration vector in the three components has estimated to be  $(7.47\pm0.46, 0.17\pm0.57, \text{ and } 3.95\pm0.47) \text{ } mm\cdot s^{-1} \cdot yr^{-1}$ . To verify the reliability and stability of our results, the entire VLBI data set was divided into four groups. The four data groups were formed as follows: all sessions were sorted and numbered from 1 to 4632 in time order, then all sessions except the sessions whose numbers were 4n+1 were included in the first group, all sessions except the sessions whose numbers were 4n+2 were included in the second group and so on, where  $n=0, 1, 2, \cdots, 1157$ . Each data group was processed in the same way. The results are listed in Table 1. The components towards the Galactic center and perpendicular to the Galactic plane show good consistency. The components in the direction along the solar motion in the Galactic plane are insignificant with respect to their formal errors. Figure 1 shows the apparent proper motion field of 295 defining sources based on the acceleration we estimated.

Table 1. The solar acceleration estimated from different data sets in the Galactic coordinate system (unit:  $mm \cdot s^{-1} \cdot ur^{-1}$ ) [9].

Number of Sess.	$X(galactic\ center)$	Y	$Z(galactic\ pole)$
3474	$7.44 \pm 0.53$	$0.63 \pm 0.67$	$3.27 \pm 0.54$
3474	$7.28 \pm 0.53$	$-0.12 \pm 0.66$	$3.93 \pm 0.53$
3474	$7.22 \pm 0.53$	$0.06 \pm 0.66$	$4.55 \pm 0.54$
3474	$7.86 \pm 0.53$	$-0.41 \pm 0.67$	$3.98 \pm 0.54$

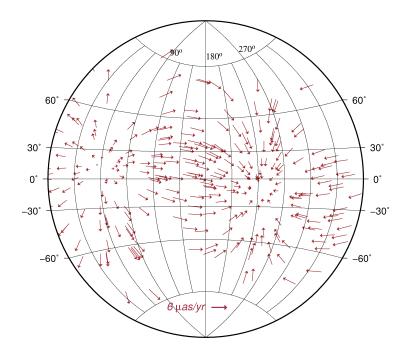


Figure 1. The apparent proper motion field of 295 defining sources based on the estimated acceleration.

#### 4. Conclusion

On the basis of the rigorous mathematical relations between the acceleration of the solar system in the Milky Way and the secular aberration drift, we estimate the acceleration by using VLBI data spanning 30 years. This method is independent of the kinematic or dynamic model of the Milky Way and also without reference to any objects in the Galaxy or the Local Standard of Rest. The acceleration vector we estimated is  $(7.47\pm0.46,\ 0.17\pm0.57,\ 3.95\pm0.47)\ mm\cdot s^{-1}\cdot yr^{-1}$  in the Galactic Cartesian coordinate system. Our estimation, which is absolutely determined relative to extragalactic radio sources with the kinematical method, has obtained an accuracy of approximately two orders of magnitude in the acceleration, the highest accuracy for current relative works.

Traditionally much of the attention paid in these studies of the secular aberration was on the Galactocentric component of the acceleration, since it was generally believed that the acceleration component in the direction normal to the Galactic plane was too small to be detected. Then the acceleration vector should nearly point to the Galactic center. This recent work, however, indicates that this vertical component is significant with its amplitude of  $4.0 \ mm \cdot s^{-1} \cdot yr^{-1}$ . Apparently this vertical acceleration should provide another meaningful parameter for the research of the Milky Way.

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